

Non-commutative measure theory and function theory

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Measure theory on the complex unit circle and analytic function theory in the unit disk, in particular the theory of Hardy spaces, are fundamentally connected. Several celebrated theorems due to P. Fatou, G. Herglotz, F. and M. Riesz and G. Szegö describe the relationship between these theories.

We will show that many of these classical results have natural extensions to the multivariate and non-commutative settings of the *full Fock space*, or *free Hardy space* of square-summable power series in several non-commuting variables and *positive non-commutative (NC) measures*. Here a (positive) NC measure is any positive linear functional on the *free disk system*, the operator system generated by the left creation operators, which act as left multiplication by the independent NC variables on the free Hardy space. We will focus on a recently established ‘NC Szegö theorem’ and its consequences. In particular, we will show that this, combined with our previously established ‘NC Fatou theorem’, provides a concrete formula for the ‘NC Radon–Nikodym derivative’ of any positive NC measure with respect to a canonical NC Lebesgue measure.