MATH 6031 Course Outline

General Information

Course Title: Functional Analysis

Course Description: Metric spaces, normed spaces, Banach spaces, and Hilbert spaces will be covered. The Baire Category theorem will be covered along with some of its consequences. The theory of linear functionals will be discussed including the Hahn-Banach theorem, dual spaces, and if time permits, weak topologies. Basic operator theory is covered including topics such as adjoints, compact operators, and spectral theory. Applications to quantum information theory will be presented throughout the course.

Credit Weight: 0.5

Academic Department: Mathematics & Statistics

Campus: University of Guelph

Semester Offering: Fall 2020

Class Schedule and Location: Weekly face-to-face meetings on Wednesdays from 430pm – 720pm in ROZH 102. We would likely move to synchronous Zoom class meetings if needed during the semester.

Instructor Information

Instructor Name: Prof. David Kribs

Instructor Email: dkribs@uoguelph.ca

Office location and office hours: Mondays 10am – 12pm over Zoom starting September 21 (make appointment via email)

Course Content

Calendar Description: Hilbert, Banach and metric spaces are covered including applications. The Baire Category theorem is covered along with its consequences such as the open mapping theorem, the principle of uniform boundedness and the

closed graph theorem. The theory of linear functionals is discussed including the Hahn-Banach theorem, dual spaces, and if time permits, weak topologies or generalized functions. Basic operator theory is covered including topics such as adjoints, compact operators, the Frechet derivative and spectral theory. A brief introduction to the concepts of measure and integration required for some of the aforementioned topics is also included.

Specific Learning Outcomes: Students in this course will learn the basic ideas and tools of functional analysis. Students will learn how to construct proofs and write and present mathematical arguments. Students will learn techniques to apply functional analysis tools, with emphasis on examples from quantum information theory.

Lecture Content:

The following are the topics that will be covered during the course, with the proviso that some changes may be made during the semester due to time limitations or other unforeseen reasons.

Chapter 1 from the course textbook (Weeks 1 and 2 likely): Review of metric and topological spaces, including equivalent metrics, Cauchy sequences and completeness, compact sets, and Baire Category Theorem and some consequences such as the Principle of Uniform Boundedness. Quantum bits (qubits) and the Bloch sphere will be introduced.

Chapter 2 from the course textbook (Weeks 3 to 6 likely): Normed and Hilbert spaces will be presented, starting with Banach spaces and equivalent norms. The theory of linear functionals will be considered, including the Hahn-Banach Theory and convex sets. Separable and entangled quantum states, and entanglement witness applications, will be discussed. Dual spaces and weak topologies will be discussed. Bases of Hilbert spaces, and Hilbert space direct sums and tensor products will be discussed.

Chapter 3 from the course textbook (Weeks 7 to 10 likely): The basic theory of linear operators on Hilbert space will be presented, beginning with the Hilbert space dual space and adjoint operators, and including the equivalence of boundedness and continuity. Special classes of operators will be discussed, including unitary operators, positive operators, projections, and normal operators. Applications to quantum algorithms and quantum measurement theory will be discussed. Basic spectral theory will be considered, including the Riesz functional calculus and the case of compact operators. Quantum density operators and mixed states will be presented as a special case.

Weeks 11 and 12: Time permitting, other topics from functional analysis or applications of the topic to quantum information theory may be presented; in particular, topics from Chapters 4 and 5 from the course textbook. Time will also be preserved near the end of the course for project presentations.

Course Resources

Required Text: J. Crann, D.W. Kribs, V. Paulsen, Functional Analysis for Quantum Information, draft textbook, 2020. Text will be distributed at the start of class.

Course Policies

Grading Policies:

- Written Assignments (2 in total, due approximately in weeks 5 and 10, 20% each): 40%
- Short In-Class Presentations (approximately 4-5 in total, each 10-15 minutes duration) on Course Material During the Semester: 25%
- Final Written Project (approximately 10 LaTex pages, due at the end of classes) and Short Project Presentation In-Class Late Semester (approximately 20 minutes duration): 35%

Course Policy on Group Work: While you may collaborate with one another or consult any reference about the general ideas on assignments, any written work must be your own. It is not permitted to ask someone for step by step guidance on assignment problem nor to copy from the work of another student or any other source.

Disclaimer For This Semester

Please note that the ongoing COVID-19 pandemic may necessitate a revision of the format of course offerings and academic schedules. Any such changes will be announced via class email. All University-wide decisions will be posted on the COVID-19 website https://news.uoguelph.ca/2019-novel-coronavirus-information/ and circulated by email.

Illness Verification For This Semester

The University will not require verification of illness (doctor's notes) for the fall 2020 or winter 2021 semesters.

Standard Statements

The following are standard statements for inclusion on all course outlines. Some departments or colleges may also elect to post this information on a common website and link to such sites in the course. However, it is strongly recommended that statements on academic misconduct and links to the academic misconduct section of the academic calendars are included on all course outlines.

E-mail Communication

As per university regulations, all students are required to check their <mail.uoguelph.ca> e-mail account regularly: e-mail is the official route of communication between the University and its students.

When You Cannot Meet a Course Requirement

When you find yourself unable to meet an in-course requirement because of illness or compassionate reasons, please advise the course instructor (or designated person, such as a teaching assistant) in writing, with your name, id#, and e-mail contact. See the undergraduate calendar for information on regulations and procedures for Academic Consideration.

Drop Date

To avoid academic penalty, courses that are one semester long must be dropped by the end of the last day of classes; in the Fall 2020 semester this day is December 4, 2020. The regulations and procedures for Dropping Courses are available in the Undergraduate Calendar.

Copies of out-of-class assignments

Keep paper and/or other reliable back-up copies of all out-of-class assignments: you may be asked to resubmit work at any time.

Accessibility

The University promotes the full participation of students who experience disabilities in their academic programs. To that end, the provision of academic accommodation is a shared responsibility between the University and the student.

When accommodations are needed, the student is required to first register with Student Accessibility Services (SAS). Documentation to substantiate the existence of a disability is required, however, interim accommodations may be possible while that process is underway.

Accommodations are available for both permanent and temporary disabilities. It should be noted that common illnesses such as a cold or the flu do not constitute a disability.

Use of the SAS Exam Centre requires students to book their exams at least 7 days in advance, and not later than the 40th Class Day.

More information: www.uoguelph.ca/sas

Academic Misconduct

The University of Guelph is committed to upholding the highest standards of academic integrity and it is the responsibility of all members of the University community – faculty, staff, and students – to be aware of what constitutes academic misconduct and to do as much as possible to prevent academic offences from occurring. University of Guelph students have the responsibility of abiding by the University's policy on academic misconduct regardless of their location of study; faculty, staff and students have the responsibility of supporting an environment that discourages misconduct. Students need to remain aware that instructors have access to and the right to use electronic and other means of detection.

Please note: Whether or not a student intended to commit academic misconduct is not relevant for a finding of guilt. Hurried or careless submission of assignments does not excuse students from responsibility for verifying the academic integrity of their work before submitting it. Students who are in any doubt as to whether an action on their part could be construed as an academic offence should consult with a faculty member or faculty advisor.

The Academic Misconduct Policy is detailed in the Undergraduate Calendar.

Recording of Materials

Presentations which are made in relation to course work—including lectures—cannot be recorded or copied without the permission of the presenter, whether the instructor, a classmate or guest lecturer. Material recorded with permission is restricted to use for that course unless further permission is granted.

Resources

The Academic Calendars are the source of information about the University of Guelph's procedures, policies and regulations which apply to undergraduate, graduate and diploma programs.