

MATH 6011 — Dynamical Systems I

Department of Mathematics and Statistics, University of Guelph

General Course Information

Instructor: [A. Willms](#), MACN 512, ext. 52736

Office Hours: TBA

Lectures: Mon., Wed., & Fri. 8:30-9:20 a.m. in GRHM 2302

Website: This course is on the web at the University's [CourseLink](#) page. It contains the information on this page as well as other information that will be updated regularly, including assignments, and posted marks.

Calendar Description

Basic theorems on existence, uniqueness and differentiability; phase space, flows, dynamical systems; review of linear systems, Floquet theory; Hopf bifurcation; perturbation theory and structural stability; differential equations on manifolds. Applications drawn from the biological, physical, and social sciences.

Learning Outcomes

- Gain a geometric intuition of dynamical systems properties.
- Understand the basic existence, uniqueness, and continuity property proofs for ordinary differential equations.
- Be able to explain the fundamental differences and relationships between discrete and continuous dynamical systems.
- Understand behaviour of systems around hyperbolic fixed points.
- Identify and analyze the generic types of bifurcations of equilibria for both discrete and continuous systems.
- Be introduced to chaos and strange attractors.

Topics

- Taylor's Theorem and the Implicit Function Theorem.
- An example of continuous and discrete dynamical systems: The logistic differential equation (DE) vs. the logistic map.
- Review of linear DEs on \mathbb{R}^n ; linear maps on \mathbb{R}^n .
- Existence and uniqueness for solutions of nonlinear DEs on \mathbb{R}^n
- Definitions: manifolds, diffeomorphisms, flows, stability, invariant sets, limit sets, etc.
- Poincaré maps and suspensions.
- Conjugacy and equivalence.
- Hyperbolic fixed points, Hartman-Grobman Theorem, Invariant (Stable) Manifold Theorem. Hyperbolic limit cycles.
- Centre Manifold Theorem, computation of the reduced system on the centre manifold.
- Structural stability and Peixoto's Theorem.
- The Smale Horseshoe.
- Codimension one bifurcations of equilibria of vector fields: saddle-node, transcritical, pitchfork, Hopf.
- Codimension one bifurcations of equilibria of maps (and periodic orbits): saddle-node, transcritical, pitchfork, flip (period doubling), Neimark-Sacker.
- Normal forms.

Texts

The following texts are useful references, they have been placed on reserve in the library. A list of topics and the relevant sections of the below text books will be on the course web page.

- Y. Kuznetsov, *Elements of Applied Bifurcation Theory*, 3rd Edition, Springer, 2004.

The second edition of Kuznetsov's book is available electronically (no downloads) through the library; but is restricted to three users at a time.

- J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag, 1983.
- L. Perko, *Differential Equations and Dynamical Systems*, 3rd Edition, Springer-Verlag, 2001. (or earlier editions)

Evaluation

| Assessment | due date | weight |
|----------------------|--|--------|
| 5 Assignments | Jan. 27, Feb. 10, Mar. 3, Mar. 17, Mar. 31 | 80% |
| Take Home Final Exam | handed out: Apr. 10; due: Apr. 24. | 20% |

Assignments

Assignments will be handed at least one week prior to their due date. They are to be completed and written up on your own. You are free to ask me questions, but collaboration with other students should be minimal, confined to discussing topics covered in class and not specific assignment questions. Late assignments will not be accepted.

University Policies

E-mail Communication

As per university regulations, all students are required to check their uoguelph.ca e-mail account regularly: e-mail is the official route of communication between the University and its students.

When You Cannot Meet a Course Requirement

When you find yourself unable to meet an in-course requirement because of illness or compassionate reasons, please advise the course instructor (or designated person, such as a teaching assistant) in writing, with your name, id#, and e-mail contact. See the Undergraduate Calendar for information on regulations and procedures for [Academic Consideration](#).

Drop Date

Courses that are one semester long must be dropped by the end of the last day of classes; two-semester courses must be dropped by the last day of classes in the second semester. The regulations and procedures for [Dropping Courses](#) are available in the Graduate Calendar.

Copies of Out-Of-Class Assignments

Keep paper and/or other reliable back-up copies of all out-of-class assignments: you may be asked to resubmit work at any time.

Accessibility

The University promotes the full participation of students who experience disabilities in their academic programs. To that end, the provision of academic accommodation is a shared responsibility between the University and the student.

When accommodations are needed, the student is required to first register with Student Accessibility Services (SAS). Documentation to substantiate the existence of a disability is required, however, interim accommodations may be possible while that process is underway.

Accommodations are available for both permanent and temporary disabilities. It should be noted that common illnesses such as a cold or the flu do not constitute a disability.

Use of the SAS Exam Centre requires students to make a booking at least 14 days in advance, and no later than November 1 (fall), March 1 (winter) or July 1 (summer). Similarly, new or changed accommodations for online quizzes, tests and exams must be approved at least a week ahead of time.

More information: www.uoguelph.ca/sas.

Academic Misconduct

The University of Guelph is committed to upholding the highest standards of academic integrity and it is the responsibility of all members of the University community – faculty, staff, and students – to be aware of what constitutes academic misconduct and to do as much as possible to prevent academic offences from occurring. University of Guelph students have the responsibility of abiding by the University's policy on academic misconduct regardless of their location of study; faculty, staff and students have the responsibility of supporting an environment that discourages misconduct. Students need to remain aware that instructors have access to and the right to use electronic and other means of detection.

Please note: Whether or not a student intended to commit academic misconduct is not relevant for a finding of guilt. Hurried or careless submission of assignments does not excuse students from responsibility for verifying the academic integrity of their work before submitting it. Students who are in any doubt as to whether an action on their part could be construed as an academic offence should consult with a faculty member or faculty advisor.

The [Academic Misconduct Policy](#) is outlined in the Graduate Calendar:

Recording of Materials

Presentations which are made in relation to course work—including lectures—cannot be recorded or copied without the permission of the presenter, whether the instructor, a classmate or guest lecturer. Material recorded with permission is restricted to use for that course unless further permission is granted.

Resources

The [Academic Calendars](#) are the source of information about the University of Guelph's procedures, policies and regulations which apply to undergraduate, graduate and diploma programs.

Disclaimer

Please note that the ongoing COVID-19 pandemic may necessitate a revision of the format of course offerings, changes in classroom protocols, and academic schedules. Any such changes will be announced via Courselink and/or class email.

This includes on-campus scheduling during the semester, mid-terms and final examination schedules. All University-wide decisions will be posted on the COVID-19 website (<https://news.uoguelph.ca/2019-novel-coronavirus-information/>) and circulated by email.

Illness

Medical notes will not normally be required for singular instances of academic consideration, although students may be required to provide supporting documentation for multiple missed assessments or when involving a large part of a course (e.g., final exam or major assignment).

COVID-19 Safety Protocols

For information on current safety protocols, follow these links:

- <https://news.uoguelph.ca/return-to-campuses/how-u-of-g-is-preparing-for-your-safe-return/>
- <https://news.uoguelph.ca/return-to-campuses/spaces/#ClassroomSpaces>

Please note, that these guidelines may be updated as required in response to evolving University, Public Health or government directives.