

University of Guelph
Department of Mathematics and Statistics
College of Engineering and Physical Sciences

MATH*2000 Proofs, Sets, and Numbers
Fall 2023
0.5 credits

1: Course Details

1.1 Calendar Description

This course exposes the student to formal mathematical proof, and introduces the theory of sets and number systems. Topics include relations and functions, number systems including formal properties of the natural numbers, integers, and the real and complex numbers. Equivalence relations and partial and total orders are introduced. The geometry and topology of the real number line and Cartesian plane are introduced. Techniques of formal proof are introduced including well-ordering, mathematical induction, proof by contradiction, and proof by construction. These techniques will be applied to fundamental theorems from linear algebra.

Prerequisite(s): 1 of [IPS*1500](#), [MATH*1080](#), [MATH*1160](#), [MATH*1200](#)

1.2 Course Description

Whereas most mathematics classes that students, going into this course, have had at university or in high school may have focused on calculating things, this course is intended as an introduction to rigorous, proof-based mathematics. It will contain an introduction to mathematical logic, which will then be applied to develop methods of proof. A mathematical proof is correct if it is logically air tight. Proof techniques will be illustrated with applications in naive set theory, elementary number theory, linear algebra, properties of the real numbers. Accurate reading and accurate writing are important in Mathematics. This will be emphasised throughout the course.

2: Instructional Support

3: Learning Resources

Students are expected to take their own notes during lectures. No lecture notes will be distributed. Written assignments will be an important part of the course that contain practice exercises and a more in depth treatment of some material. Assignments will be posted on course link, so will be the material of the lab sessions.

The **required textbook** for this course is

Ulrich Daepf, Pamela Gorkin, *“Reading, Writing, and Proving. A Closer Look at Mathematics”*, 2nd Edition, Springer, New York, 2011

This book is available in electronic format (pdf), through the university library. There is also an earlier edition from 2003 (also available in electronic format from the library). We will follow this book closely, and **students will be required and expected to read it.**

The following books are recommended as further reading for interested students:

George Polya, *“How to Solve it: A New Aspect of Mathematical Method”*, Princeton Science Library, Princeton University Press, 2014

This is a classic text on mathematical problem solving in general (not specific to the topic of our course). Our primary text book follows an approach to mathematics that has been outlined in this book. The above is the current edition; there have been many previous ones going back to 1945.

Daniel J Velleman, *“How to Prove it: A structured approach”*, 2nd edition, Cambridge University Press, 2005

This book covers very similar material as the primary textbook (as well as other material) but sometimes in a different order. It is more technical. There are also one earlier (1994) and one newer (2019) edition.

Some students might also find the following interesting, both of which are available in electronic form from the library:

Steven G. Krantz, *“The Proof is in the Pudding. The Changing Nature of Mathematical Proof”*, Springer, 2010, specifically chapter 1 “What is Proof and Why?”

Martin Aigner, Gunther M Ziegler. *“Proofs from THE BOOK”*, 5th ed. Springer, 2014

4: Learning Outcomes

1. Critical and logical thinking: Students will learn how to critically examine logical statements for their veracity, and how to develop logically correct argumentation.

2. Concepts of mathematical proofs and proof techniques: These include direct proof, proof by contradiction, proof in cases, mathematical induction; also the role of counter examples in mathematics will be discussed.
3. Independent learning: Reading the required text is mandatory. This may expose students to additional aspects that are not explicitly covered in class, but may be used in later chapters.
4. Mathematical communication: In Mathematics, probably more than in any other discipline, accurate language is paramount. Even small errors or ambiguities can render a statement meaningless, false, or incomprehensible. Accurate reading and writing of mathematics are therefore practised and emphasised in this course throughout the semester.

5: Teaching and Learning Activities

Method of instruction

The course follows a traditional lecture model and includes written assignments to practise the material covered in the lectures. Labs are held for further enforcement of the material from the lectures, additional applications, etc.

Accurate reading is an important pre-requisite for accurate writing. Students will be expected to read every week the assigned chapters in the textbook that pertain to the material covered in class. In the lectures I will present an abridged version of the material in the textbook, emphasising the more challenging points. Whereas the textbook is written in rather colloquial vernacular, intended for leisurely independent reading, the presentation of the material in class will be more formal and technical. This will familiarise students with two approaches to reading and writing mathematics that complement each other.

Written assignments are an important part of this course, not only for evaluation but probably more importantly for learning. Mathematics is not a spectator sport. The written assignments are a hands-on component of the course in which students will practice mathematics. Theorems that are proved in the assignments are part of the course curriculum and will have the same status as theorems proved in class. The written assignments will require properly formulated answers and rigorous, precise and complete argumentation.

As per the academic calendar, students should expect to spend 10-12 hrs/wk for their course work, including lectures, labs, readings and assignments.

Lecture Topics (tentative):

1. Introductory Remarks
2. Logically Speaking: Sentential Logic
3. The Contrapositive and the Converse
4. Set Notation and Quantifiers
5. Some Proof Techniques
6. Sets
7. Operations on Sets
8. More Sets
9. The Power Set and the Cartesian Product
10. Relations
11. Partitions
12. Order in the Reals
13. Completeness of \mathbb{R}
14. Functions: Domain and Range
15. Functions: One-to-one, and Onto (Injective, Surjective, and Bijective)
16. Inverses
17. Images and Inverse Images
18. Mathematical Induction

Time Permitting:

- 19. Sequences
- 20. Convergence of Sequences
- 21. Additional examples and applications

6: Assessments

6.1 Marking Schemes & Distributions

Final grades will be determined based on the following:

Best 8 out of 9 weekly assignments	56% [7% per assignment]
Final exam	44%

6.2 Assessment Details

Written assignments will be distributed at least one week before the due date. Solutions will be posted after the assignments have been due. We do recognize that some students learn best in groups. Therefore, assignments can be submitted by groups of up to 4 students, although we strongly recommend that every student works on all assignment questions on their own, as this will be the best approach to learning and the preparation for the final exam. No late assignments will be accepted.

Schedule of assignments.

Assignment 1	Thursday, Sep 21
Assignment 2	Thursday, Sep 28
Assignment 3	Thursday, Oct 5
Assignment 4	Thursday, Oct 19
Assignment 5	Thursday, Oct 26
Assignment 6	Thursday, Nov 2
Assignment 7	Thursday Nov 9
Assignment 8	Thursday, Nov 16
Assignment 9	Thursday, Nov 23

Final Exam: FRI, Dec 8, 2023, 8:30-10:30 (room TBA).

This will be a closed book exam to which each student will be permitted to bring one letter sized sheet of their own hand written notes as the only aid. The Final Exam will be individual work.

Grades and interpretation of grades. The normal grading system that is in use by the university applies. Grading system and interpretation of grades are described in detail in the undergraduate calendar, under

<https://calendar.uoguelph.ca/undergraduate-calendar/undergraduate-degree-regulations-procedures/grades/>

7: Additional University Statements

Email Communication

As per university regulations, all students are required to check their <uoguelph.ca> e-mail account regularly: e-mail is the official route of communication between the University and its students.

When You Cannot Meet a Course Requirement

When you find yourself unable to meet an in-course requirement because of illness or compassionate reasons, please advise the course instructor (or designated person, such as a teaching assistant) in writing, with your name, id#, and e-mail contact. See the Undergraduate Calendar for information on regulations and procedures for [Academic Consideration](#).

Drop Date

Courses that are one semester long must be dropped by the end of the last day of classes; two-semester courses must be dropped by the last day of classes in the second semester. The regulations and procedures for [Dropping Courses](#) are available in the Undergraduate Calendar.

Copies of Out-Of-Class Assignments

Keep paper and/or other reliable back-up copies of all out-of-class assignments: you may be asked to resubmit work at any time.

Accessibility

The University promotes the full participation of students who experience disabilities in their academic programs. To that end, the provision of academic accommodation is a shared responsibility between the University and the student.

When accommodations are needed, the student is required to first register with Student Accessibility Services (SAS). Documentation to substantiate the existence of a disability is required, however, interim accommodations may be possible while that process is underway.

Accommodations are available for both permanent and temporary disabilities. It should be noted that common illnesses such as a cold or the flu do not constitute a disability.

Use of the SAS Exam Centre requires students to make a booking at least 14 days in advance, and no later than November 1 (fall), March 1 (winter) or July 1 (summer). Similarly, new or changed accommodations for online quizzes, tests and exams must be approved at least a week ahead of time.

More information: www.uoguelph.ca/sas

Academic Misconduct

The University of Guelph is committed to upholding the highest standards of academic integrity and it is the responsibility of all members of the University community – faculty, staff, and students – to be aware of what constitutes academic misconduct and to do as much as

possible to prevent academic offences from occurring. University of Guelph students have the responsibility of abiding by the University's policy on academic misconduct regardless of their location of study; faculty, staff and students have the responsibility of supporting an environment that discourages misconduct. Students need to remain aware that instructors have access to and the right to use electronic and other means of detection.

Please note: Whether or not a student intended to commit academic misconduct is not relevant for a finding of guilt. Hurried or careless submission of assignments does not excuse students from responsibility for verifying the academic integrity of their work before submitting it. Students who are in any doubt as to whether an action on their part could be construed as an academic offence should consult with a faculty member or faculty advisor.

The [Academic Misconduct Policy](#) is outlined in the Undergraduate Calendar.

Recording of Materials

Presentations which are made in relation to course work—including lectures—cannot be recorded or copied without the permission of the presenter, whether the instructor, a classmate or guest lecturer. Material recorded with permission is restricted to use for that course unless further permission is granted.

Resources

The [Academic Calendars](#) are the source of information about the University of Guelph's procedures, policies and regulations which apply to undergraduate, graduate and diploma programs.

Disclaimer

Please note that the ongoing COVID-19 pandemic may necessitate a revision of the format of course offerings, changes in classroom protocols, and academic schedules. Any such changes will be announced via Courselink and/or class email.

This includes on-campus scheduling during the semester, mid-terms and final examination schedules. All University-wide decisions will be posted on the COVID-19 website (<https://news.uoguelph.ca/2019-novel-coronavirus-information/>) and circulated by email.

Illness

Medical notes will not normally be required for singular instances of academic consideration, although students may be required to provide supporting documentation for multiple missed assessments or when involving a large part of a course (e.g., final exam or major assignment).

COVID-19 Safety Protocols

For information on current safety protocols, follow these links:

- <https://news.uoguelph.ca/return-to-campus/how-u-of-g-is-preparing-for-your-safe-return/>
- <https://news.uoguelph.ca/return-to-campus/spaces/#ClassroomSpaces>

Please note, that these guidelines may be updated as required in response to evolving University, Public Health or government directives.