MATH*6031 Course Outline

Course Title: Functional Analysis

Credit Weight: 0.5

Restriction: Credit may be obtained for only one of MATH*4220 or MATH*6031

Academic Department: Mathematics & Statistics

Campus: University of Guelph

Semester Offering: Fall 2024

Instructor Information

Instructor Name:

Instructor Email:

Office hours (via Zoom):

Course Content and Delivery

Class Schedule and Delivery Method:

Course Materials:

- **Course textbook (required and PDF posted on Courselink)**: D.W. Kribs, V. Paulsen, *"Functional Analysis for Quantum Information,"* draft textbook, 2024.
- **Lecture notes**: hand-written lecture notes will be posted on Courselink before each class.
- **Lecture videos:** Some of the class material may be posted as recorded videos on Courselink and this will be announced in class.
- **Supplementary course resources**: There are many Functional Analysis textbooks that can be found online or in the library, which would complement the course textbook. Two of the classics: W. Rudin, *"Functional Analysis,"* McGraw-Hill, Second Ed., 1991; and, J. Conway, *"A Course in Functional Analysis,"* Springer-Verlag, Second Ed., 2007.
- **Other course materials:** such as assignments and solutions will be announced in class or via group email and will be posted on Courselink.

Course Content

Calendar Description: Hilbert, Banach and metric spaces are covered including applications. The Baire Category theorem is covered along with its consequences such as the open mapping theorem, the principle of uniform boundedness and the closed graph theorem. The theory of linear functionals is discussed including the Hahn-Banach theorem, dual spaces, and if time permits, weak topologies or generalized functions. Basic operator theory is covered including topics such as adjoints, compact operators, the Frechet derivative and spectral theory. A brief introduction to the concepts of measure and integration required for some of the aforementioned topics is also included.

Specific Learning Outcomes and Alignment of Assessments: Students in this course will learn the basic ideas and tools of functional analysis. Students will learn how to construct proofs and write and present mathematical arguments. Students will learn techniques to apply functional analysis tools and glimpse the wide applicability of the subject in the mathematical sciences. The assessments described below, including the written assignments, presentations, and expository research paper, will be aligned to test for these learning outcomes.

Special Note for This Semester: Some of the concepts listed above will be presented along with applications to quantum computing and quantum information theory, as examples of the far-reaching applicability of the subject, but no prior knowledge of those subjects is necessary to take the course.

Lecture Content: The following are the topics that will be covered during the course and the tentative timeline, with the proviso that some changes may be made during the semester due to time limitations or other unforeseen reasons (any such adjustments will be announced in class).

Chapter 1 from the course textbook (Weeks 1 and 2): Review of metric and topological spaces, including equivalent metrics, Cauchy sequences and completeness, compact sets, and Baire Category Theorem and some consequences such as the Principle of Uniform Boundedness. Quantum bits (qubits) and the Bloch sphere will be introduced.

Chapter 2 from the course textbook (Weeks 3 to 6): Normed and Hilbert spaces will be presented, starting with Banach spaces and equivalent norms. The theory of linear functionals will be considered, including the Hahn-Banach Theory and convex sets. Separable and entangled quantum states, and entanglement witness applications, will be discussed. Dual spaces and weak topologies will be discussed. Bases of Hilbert spaces, and Hilbert space direct sums and tensor products will be discussed.

Chapter 3 from the course textbook (Weeks 7 to 10): The basic theory of linear operators on Hilbert space will be presented, beginning with the Hilbert space dual

space and adjoint operators, and including the equivalence of boundedness and continuity. Special classes of operators will be discussed, including unitary operators, positive operators, projections, and normal operators. Applications to quantum algorithms and quantum measurement theory will be discussed. Basic spectral theory will be considered, including the Riesz functional calculus and the case of compact operators. Quantum density operators and mixed states will be presented as a special case.

Weeks 11 and 12: The last classes will be used to cover remaining material listed above if needed. Time permitting, other topics from functional analysis may be presented; in particular, topics from Chapters 4 and 5 from the course textbook.

Course Grading Policies

Grading Policies:

- Written Assignments (2 in total, due approximately in weeks 6 and 11, with specific due dates given during the semester, 25% each): 50%
- Two 25-minute Presentations on course material during the semester (topics and lecture dates to be determined through discussion with Prof. Kribs during the semester): 25%
- Final Written Project (approximately 10 LaTex pages, on a topic in Functional Analysis or its Applications, to be determined in discussion with Prof. Kribs during the semester). Due Friday, December 6 at 11:59pm: 25%

Course Policy on Group Assignment Work and Late Submission Policy: While you may collaborate with one another or consult any reference about the general ideas on assignments, any written work must be your own. It is not permitted to ask someone for step-by-step guidance on assignment problem nor to copy from the work of another student or any other source. Please contact the instructor if you cannot make a submission deadline for one of the written assignments or the written project. If needed, an extension of up to five additional days will be granted.

University Standard Statements

Academic Integrity

The University of Guelph is committed to upholding the highest standards of academic integrity and it is the responsibility of all members of the University community – faculty, staff, and students – to be aware of what constitutes academic misconduct and to do as much as possible to prevent academic offences from occurring. University of Guelph students have the responsibility of abiding by the University's policy on academic

misconduct regardless of their location of study; faculty, staff and students have the responsibility of supporting an environment that discourages misconduct. Students need to remain aware that instructors have access to and the right to use electronic and other means of detection.

Please note: Whether or not a student intended to commit academic misconduct is not relevant for a finding of guilt. Hurried or careless submission of assignments does not excuse students from responsibility for verifying the academic integrity of their work before submitting it. Students who are in any doubt as to whether an action on their part could be construed as an academic offence should consult with a faculty member or faculty advisor.

The Academic Misconduct Policy is outlined in the Undergraduate Calendar.

Accessibility

The University promotes the full participation of students who experience disabilities in their academic programs. To that end, the provision of academic accommodation is a shared responsibility between the University and the student.

When accommodations are needed, the student is required to first register with Student Accessibility Services (SAS). Documentation to substantiate the existence of a disability is required, however, interim accommodations may be possible while that process is underway.

Accommodations are available for both permanent and temporary disabilities. It should be noted that common illnesses such as a cold or the flu do not constitute a disability.

Use of the SAS Exam Centre requires students to make a booking at least 10 business days in advance, and no later than the first business day in November, March or July as appropriate for the semester. Similarly, new or changed accommodations for online quizzes, tests and exams must be approved at least a week ahead of time.

More information: www.uoguelph.ca/sas.

Accommodation of Religious Obligations

If you are unable to meet an in-course requirement due to religious obligations, please email the course instructor within two weeks of the start of the semester to make alternate arrangements.

See the Academic calendar for information on regulations and procedures for Academic Accommodation of Religious Obligations.

Copies of Out-Of-Class Assignments

Keep paper and/or other reliable back-up copies of all out-of-class assignments: you may be asked to resubmit work at any time.

Drop Date

Courses that are one semester long must be dropped by the end of the last day of classes; two-semester courses must be dropped by the last day of classes in the second semester. The regulations and procedures for Dropping Courses are available in the Undergraduate Calendar.

Email Communication

As per university regulations, all students are required to check their <uoguelph.ca> e-mail account regularly: e-mail is the official route of communication between the University and its students.

Health and Wellbeing

The University of Guelph provides a wide range of health and wellbeing services at the Vaccarino Centre for Student Wellness. If you are concerned about your mental health and not sure where to start, connect with a Student Wellness Navigator who can help develop a plan to manage and support your mental health or check out our mental wellbeing resources. The Student Wellness team are here to help and welcome the opportunity to connect with you.

Illness

Medical notes will not normally be required for singular instances of academic consideration, although students may be required to provide supporting documentation for multiple missed assessments or when involving a large part of a course (e.g., final exam or major assignment).

Recording of Materials

Presentations which are made in relation to course work—including lectures—cannot be recorded or copied without the permission of the presenter, whether the instructor, a classmate or guest lecturer. Material recorded with permission is restricted to use for that course unless further permission is granted.

Resources

The Academic Calendars are the source of information about the University of Guelph's procedures, policies and regulations which apply to undergraduate, graduate and diploma programs.

When You Cannot Meet a Course Requirement

When you find yourself unable to meet an in-course requirement because of illness or compassionate reasons, please advise the course instructor (or designated person, such as a teaching assistant) in writing, with your name, id#, and e-mail contact. See the Undergraduate Calendar for information on regulations and procedures for Academic Consideration.