UNIVERSITY of GUELPH

MATH*3510 Biomathematics

Winter 2024

Section: C01

Department of Mathematics and Statistics

Credit Weight: 0.50

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1 Course Details

1.1 Calendar Description

This course will convey the fundamentals of applying mathematical modelling techniques to understanding and predicting the dynamics of biological systems. Students will learn the development, analysis, and interpretation of biomathematical models based on discrete-time and continuous-time models. Applications may include examples from population biology, ecology, infectious diseases, microbiology, and genetics.

Pre-Requisites: MATH*2270, (ENGG*1500 or MATH*1160)

1.2 Course Description

The course is divided into two main topics: discrete processes in biology and continuous processes and ordinary differential equations. The focus of the course will be on applying mathematical modeling techniques to analyzing biological systems.

1.3 Delivery Method

Synchronous in-person lectures. Readings will be made available before each lecture; lectures will be devoted to a summary of the material and opportunities to work through and discuss problems based on the material.

1.4 Timetable

Mondays, Wednesdays, Fridays 10:30 am – 11:20 am, MINS 017

The timetable is subject to change. Please see WebAdvisor for the latest information.

1.5 Final Exam

Monday, April 22, 8:30 am – 10:30 am, Location TBA

Exam time and location are subject to change. Please see WebAdvisor for the latest information.

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2 Instructional Support

2.1 Instructional Support Team

Instructor:	Christopher van Bommel
Email:	cvanbomm@uoguelph.ca
Telephone:	TBA
Office:	TBA
Office Hours:	Mondays 1:30 – 2:50 pm and Wednesdays 11:40 am – 12:20 pm, or by appointment

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3 Learning Resources

3.1 Recommended Textbooks

1. Leah Edelstein-Keshet, Mathematical Models in Biology, 2005. Published by SIAM. (Physical copies are available from the bookstore or electronic copies are available on the SIAM website.)

2. Linda J.S. Allen, An Introduction to Mathematical Biology, 2007, Pearson Prentice Hall. (Out of print.)

3.2 Additional Resources Posted to CourseLink

1. Pre-lecture summaries of content, available approximately a week before the lecture.
2. Lecture slides, available approximately 24 hours before the lecture.
3. Worked examples, available approximately 24 hours following the lecture.
4. Recommended practice problems, available approximately 24 hours following the lecture.
5. Previous tests, available within 1-2 weeks of the assessments.

4 Learning Outcomes

4.1 Course Learning Outcomes

This course aims to develop modelling and simulation methods and consider their applications to biological phenomena. Knowledge of differential equations is required. By the end of the course, students are expected to be able to:

- Solve difference equations and systems.
- Analyze solutions of differential equations and systems qualitatively.
- Construct, evaluate, and interpret applied biomathematics models.
- Work collaboratively with peers on a research project.
- Present a biomathematics model.

5 Teaching and Learning Activities

Date	Topic
Jan 8	**Ch 1: The Theory of Linear Difference Equations Applied to Population Growth** 1.1 Biological Models Using Difference Equations (Cell Division)
Jan 10	1.1 Biological Models Using Difference Equations (An Insect Population) 1.2 Propagation of Annual Plants
Jan 12	1.3 Systems of Linear Difference Equations 1.4 A Linear Algebra Review
Jan 15	1.5 Will Plants Be Successful? 1.6 Qualitative Behavior of Solutions to Linear Difference Equations
Jan 15	1.7 The Golden Mean Revisited 1.9 Related Applications to Similar Problems
Jan 19	**Ch 2: Nonlinear Difference Equations** 2.1 Recognizing a Nonlinear Difference Equation 2.2 Steady States, Stability, and Critical Parameters
Jan 22	2.3 The Logistic Difference Equation
Jan 24	2.5 Graphical Methods for First-Order Equations 2.6 A Word about the Computer
Jan 26	2.7 Systems of Nonlinear Difference Equations 2.8 Stability Criteria for Second-Order Equations **Ch 3: Applications of Nonlinear Difference Equations to Population Biology**
Jan 29	3.1 Density Dependence in Single-Species Populations 3.2 Two-Species Interactions: Host-Parasitoid Systems
Jan 31	3.3 The Nicholson-Bailey Model
Feb 2	**Ch 4: An Introduction to Continuous Models** 4.1 Warmup Examples: Growth of Microorganisms

Feb 5	4.2 Bacterial Growth in a Chemostat 4.3 Formulating a Model
Feb 7	4.4 A Saturating Nutrient Consumption Rate 4.5 Dimensional Analysis of the Equations 4.6 Steady-State Solutions
Feb 9	**Test 1 (Discrete Process in Biology)**
Feb 12	4.7 Stability and Linearization 4.9 When Is a Steady State Stable? 4.10 Stability of Steady States in the Chemostat
Feb 14	**Ch 5: Phase-Plane Methods and Qualitative Solutions** 5.1 First-Order ODEs: A Geometric Meaning
Feb 16	5.2 Systems of Two First-Order ODEs
	Reading Week (February 19 – 23)
Feb 26	5.4 The Direction Field 5.5 Nullclines: A More Systematic Approach
Feb 28	5.6 Close to the Steady States 5.7 Phase-Plane Diagrams of Linear Systems (Real Eigenvalues)
Mar 1	5.7 Phase-Plane Diagrams of Linear Systems (Complex Eigenvalues) 5.8 Classifying Stability Characteristics
Mar 4	5.9 Global Behavior from Local Information **Ch 6: Applications of Continuous Models to Population Dynamics**
Mar 6	6.1: Models for Single-Species Populations (Malthus Model, Logistic Growth)
Mar 8	40th Class Day 6.1 Models for Single-Species Populations (Allee Effect, Other Assumptions; Gompertz Growth in Tumors) **Ch 7: Models for Molecular Events**
Mar 11	7.1 Michaelis-Menten Kinetics 7.2 The Quasi-Steady-State Assumption
Mar 13	7.4 A Quick, Easy Derivation of Sigmoidal Kinetics **Ch 8: Limit Cycles, Oscillations, and Excitable Systems**
Mar 15	**Test 2 (Continuous Processes and Ordinary Differential Equations)**
Mar 18	8.1 Nerve Conduction, the Action Potential, and the Hodgkin-Huxley Equations
Mar 20	8.1 Nerve Conduction, the Action Potential, and the Hodgkin-Huxley Equations
Mar 22	8.2 Fitzhugh's Analysis of the Hodgkin-Huxley Equations
Mar 25	8.3 The Poincaré-Bendixson Theory
Mar 27	8.4 The Case of the Cubic Nullclines
Mar 29	**Good Friday – No Classes**
Apr 1	**Project Presentations (Teams 1 & 2)**
Apr 3	**Project Presentations (Teams 3 & 4)**
Apr 5	**Project Presentations (Teams 5 & 6)**
Apr 8	8.5 The Fitzhugh-Nagumo Model for Neural Impulses

6 Assessments

6.1 Marking Schemes & Distributions

Assessment	Weight	Dates
Homework (Best 5 out of 7)	25%	#1: Jan 19 (11:59 pm) #2: Jan 26 (11:59 pm) #3: Feb 2 (11:59 pm) #4: Feb 16 (11:59 pm) #5: Mar 1 (11:59 pm) #6: Mar 8 (11:59 pm) #7: Mar 22 (11:59 pm)
Tests	25%	Test 1: Feb 9 (In Class) Test 2: Mar 15 (In Class)
Project	25%	Topic: Jan 31 Bibliography: Feb 14 Abstract: Mar 6 Report: Mar 20 Slides: Mar 28 Presentations: Apr 1-5 Individual Reflection: Apr 8
Exam	25%	April 22, 8:30 am – 10:30 am

6.2 Homework

Each homework will be posted about a week before the due date and is due at 11:59 pm (ET) on the date indicated. Solutions to all homework assignments should be clearly written or typed in complete sentences, with each step fully explained to earn full marks. Students may discuss homework problems provided that each student independently writes a solution to the problem and collaborators are indicated for each problem. You may consult resources to aid your completion of the homework, both those provided as part of the course and those external to the course, provided you do not explicitly search for homework problems and their solutions. Homework is to be uploaded to Gradescope following the instructions provided. Since only the best five of seven homework assignments will count towards your final grade, there will typically be no further accommodations for missing homework assignments.

6.3 Tests

Test 1 will be held Friday, February 9 in class and will cover Discrete Process in Biology (Chapters 1 – 3). Test 2 will be held Friday, March 15 in class and will cover Continuous Processes and Ordinary Differential Equations (Chapters 4 – 6). Students unable to write a test will be provided the opportunity to arrange a make-up test at a time mutually agreed upon. If a

make-up test is unable to be arranged, the weight of the missed test will be shifted to the final exam.

6.4 Project

Students will be divided (randomly) into teams to study a biomathematics model not presented in the course, in particular its construction, analysis, and interpretation. The project will consist of the following milestones:

- January 31: Select Topic and Post to the CourseLink Discussion Forum. Topics can be drawn from the textbook or journal articles.
- February 14: Bibliography – Provide a list of 3-5 references you plan to consult in studying this model, and submit it to the corresponding Dropbox folder.
- March 6: Abstract – Provide a summary of your planned report/presentation (approximately 150 words) that identifies the biomathematics model you have chosen to study, what it is used for, and what conclusions can be drawn from it. The abstract should be posted to the CourseLink Discussion Forum.
- March 20: Report – Provide a 2–4-page summary of the materials in your bibliography and other materials used to study your model. It should cover what situation is being modeled, what assumptions are being made, how the equations are set up and solved, what analysis is performed on the model, and what conclusions are made and how they apply to the situation being considered. The full details of the calculations do not need to be included. Reports should be submitted to the corresponding Dropbox folder.
- March 26: Slides – Visual aids to accompany your presentation, which should be submitted to the corresponding Dropbox folder.
- April 1-5: Presentation – 15 minutes plus 5 minutes for questions from the audience. The presentation forms a mini-lecture on your chosen biomathematics model and should cover what is being modeled, what equations are used to model it and what they mean, what mathematical analysis is performed on the model (highlight results rather than the calculations), and how these results are interpreted in the context of the situation being modeled.
- April 8: Individual Reflection – A maximum one-page discussion of what you gained from the project and how you worked with your other teammates. Submitted **individually by each team member** to the appropriate Dropbox folder.

Rubric

- 10 marks for the report:
    - 2 marks for overall organization and clarity
    - 2 marks for identifying the use of the model and the assumptions being used
    - 2 marks for discussing how the model is solved
    - 2 marks for discussing the conclusions from the model and their importance
    - 2 marks for linking the model to course material
- 10 marks for the presentation:
    - 4 marks for its organization (pace and timing, slide density, use of visuals when appropriate)

    - 4 marks for clarity (ability to communicate mathematical content, highlighting the main concepts, logical order of material and transitions)
    - 2 marks for responses to queries from the audience
- 5 marks for the other milestones:
    - 1 mark for posting your topic by the deadline.
    - 1 mark for an appropriate bibliography.
    - 1 mark for posting an abstract by the deadline that provides a clear summary of the model to be covered in your report and presentation.
    - 1 mark for submitting visual aids by the deadline.
    - 1 mark for your reflection.

6.5 Exam

The exam will be comprehensive, with a greater emphasis on Chapters 7 & 8. Students unable to write the final exam due to extenuating circumstances must apply to their program councillor to have their exam deferred within five days of the exam.

6.6 Grading

We will aim to return the results of homework and tests within 5 business days.

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7 University Statements

7.1 Email Communication

As per university regulations, all students are required to check their <uoguelph.ca> e-mail account regularly: e-mail is the official route of communication between the University and its students.

7.2 When You Cannot Meet a Course Requirement

When you find yourself unable to meet an in-course requirement because of illness or compassionate reasons, please advise the course instructor (or designated person, such as a teaching assistant) in writing, with your name, id#, and e-mail contact. See the Undergraduate Calendar for information on regulations and procedures for Academic Consideration.

7.3 Drop Date

Courses that are one semester long must be dropped by the end of the last day of classes; two-semester courses must be dropped by the last day of classes in the second semester. The regulations and procedures for Dropping Courses are available in the Undergraduate Calendar.

7.4 Copies of Out-Of-Class Assignments

Keep paper and/or other reliable back-up copies of all out-of-class assignments: you may be asked to resubmit work at any time.

7.5 Accessibility

The University promotes the full participation of students who experience disabilities in their academic programs. To that end, the provision of academic accommodation is a shared responsibility between the University and the student.

When accommodations are needed, the student is required to first register with Student Accessibility Services (SAS). Documentation to substantiate the existence of a disability is required, however, interim accommodations may be possible while that process is underway.

Accommodations are available for both permanent and temporary disabilities. It should be noted that common illnesses such as a cold or the flu do not constitute a disability.

Use of the SAS Exam Centre requires students to make a booking at least 10 business days in advance, and no later than the first business day in November, March or July as appropriate for the semester. Similarly, new or changed accommodations for online quizzes, tests and exams must be approved at least a week ahead of time.

More information: www.uoguelph.ca/sas.

7.6 Academic Integrity

The University of Guelph is committed to upholding the highest standards of academic integrity and it is the responsibility of all members of the University community – faculty, staff, and students – to be aware of what constitutes academic misconduct and to do as much as possible to prevent academic offences from occurring. University of Guelph students have the responsibility of abiding by the University's policy on academic misconduct regardless of their location of study; faculty, staff and students have the responsibility of supporting an environment that discourages misconduct. Students need to remain aware that instructors have access to and the right to use electronic and other means of detection.

Please note: Whether or not a student intended to commit academic misconduct is not relevant for a finding of guilt. Hurried or careless submission of assignments does not excuse students from responsibility for verifying the academic integrity of their work before submitting it. Students who are in any doubt as to whether an action on their part could be construed as an academic offence should consult with a faculty member or faculty advisor.

The Academic Misconduct Policy is outlined in the Undergraduate Calendar.

7.7 Recording of Materials

Presentations which are made in relation to course work—including lectures—cannot be recorded or copied without the permission of the presenter, whether the instructor, a classmate or guest lecturer. Material recorded with permission is restricted to use for that course unless further permission is granted.

7.8 Resources

The Academic Calendars are the source of information about the University of Guelph's procedures, policies and regulations which apply to undergraduate, graduate and diploma programs.

7.9 Disclaimer

Please note that the ongoing COVID-19 pandemic may necessitate a revision of the format of course offerings, changes in classroom protocols, and academic schedules. Any such changes will be announced via Courselink and/or class email.

This includes on-campus scheduling during the semester, mid-terms and final examination schedules. All University-wide decisions will be posted on the COVID-19 website (https://news.uoguelph.ca/2019-novel-coronavirus-information/) and circulated by email.

7.10 Illness

Medical notes will not normally be required for singular instances of academic consideration, although students may be required to provide supporting documentation for multiple missed assessments or when involving a large part of a course (e.g., final exam or major assignment).

7.11 Accommodation of Religious Obligations

If you are unable to meet an in-course requirement due to religious obligations, please email the course instructor within two weeks of the start of the semester to make alternate arrangements.

See the Academic calendar for information on regulations and procedures for Academic Accommodation of Religious Obligations.

7.12 Health and Wellbeing

The University of Guelph provides a wide range of health and wellbeing services at the Vaccarino Centre for Student Wellness. If you are concerned about your mental health and not sure where to start, connect with a Student Wellness Navigator who can help develop a plan to manage and support your mental health or check out our mental wellbeing

resources. The Student Wellness team are here to help and welcome the opportunity to connect with you.