2013 Southwestern Ontario Graduate Mathematics Conference

June 4-5, 2013

UNIVERSITY of GUELPH

2013 Southwestern Ontario Graduate Mathematics Conference

Schedule of Talks

Tuesday, June 4, 2013.

8:45 **Registration and Breakfast**

9:15 **Jeremy Levick**, University of Guelph.
The Four-Dimensional Perfect Mirsky Conjecture.

9:45 **Colin Phipps**, University of Waterloo.
Investigating the role of interstitial fluid pressure on angiogenic behaviour in solid tumours.

10:15 **Clarence D. Kalitsi**, Brock University.
Approximate Sampling Distributions of the Parameter Estimators in the AR(1)-model.

10:45 **Coffee Break**

11:00 **Chris Bauch (Keynote Speaker)**, University of Guelph.
How mathematics can help explain vaccine scares and associated disease dynamics.

12:00 **Lunch**

1:00 **Vivian Akpene Apety**, Brock University.
Infinitesimal Symmetries of a $(1+1)$*-dimensional Integrable Hydrodynamic-type System.*

1:30 **Vardayani Ratti**, University of Guelph.
A mathematical model of honeybees and their diseases.

2:00 **Rasha Al Jamal**, University of Waterloo.
Control of the Kuramoto-Sivashinsky Equation.

2:30 **Coffee Break**

2:45 **Chris Plyley**, University of Western Ontario.
Polynomial Identities and the Duality Theorem.

3:15 **Lauren DeDieu**, McMaster University.
λ*-Harmonious Colouring.*

3:45 **Cameron McGuinness**, University of Guelph.
An Introduction to Monte Carlo Tree Search.

Wednesday, June 5, 2013.

8:45 **Registration and Snacks**

9:15 **Kazi Rahman**, University of Guelph.
Dual species model for spatially structured population.

9:45 **Richard Kohar**, Royal Military College.
Optimization of a Wireless Channel.

10:15 **Nick Rogers**, McMaster University.
Compelling Interactive Simulations: Stories from the Front of Graduate Teaching and Research in Partial Differential Equations.

10:45 **Coffee Break**

11:00 **Matthew Demers (Keynote Speaker)**, University of Guelph.
Fractal Attractors and Contractive Iterated Function Systems.

12:00 **Lunch Break**

1:00 **Rastko Anicic**, University of Waterloo.
Mathematical Modelling of Graphene Structures.

1:30 **Christiane Burton**, University of Western Ontario.
Signal and noise model of a novel energy-dependent x-ray angiographic technique.

2:00 **Kenneth Blahut**, Ryerson University.
Modeling the in vitro spread of hepatitis C virus infections.

2:30 **Coffee Break**

2:45 **Amenda Chow**, University of Waterloo.
Control of Hysteresis in the Landau-Lifshitz Equation.

3:15 **Monica Wong**, University of Guelph.
Parsimonious Biclustering of Gene Expression Data with Applications in Nutrigenomics.

3:45 **Brian Franczak**, University of Guelph.
Model-based clustering and classication via mixtures of shifted asymmetric Laplace (SAL) distributions.

2013 Southwestern Ontario Graduate Mathematics Conference (SOGMC'13)

Abstracts

Rasha Al Jamal, University of Waterloo.
Control of the Kuramoto-Sivashinsky Equation.

The Kuramoto-Sivashinsky (KS) equation is a nonlinear partial differential equation that is first-order in time and fourth-order in space. It models reaction-diffusion systems and is related to various pattern formation phenomena where turbulence or chaos appear. For instance, it models long wave motions of the liquid film over a vertical plane. For certain parameter values of interest, this equation is unstable. This is shown by analyzing the stability of the linearized system and showing that the nonlinear C0-semigroup corresponding to the nonlinear system is Frechet differentiable. There are a number of papers establishing the stabilization of this equation via boundary control. In this talk, we consider distributed control with a single control variable for the KS equation with periodic boundary conditions. First, we show that stabilizing the linearized KS equation implies local exponential stability of the KS equation. This is done by establishing Frechet differentiability of the associated semigroup and showing that it is equal to the semigroup generated by the linearization of the equation. Next, we construct a single input-feedback control that locally exponentially stabilizes the KS equation. Finally, we control the KS equation from one equilibrium solution to another.

Rastko Anicic, University of Waterloo.
Mathematical Modelling of Graphene Structures.

Graphene is a new material, which promises to revolutionize the semiconductor industry. The properties of this two dimensional allotrope of carbon has applications in THz radiation receivers, bio-sensors, transistors, ultra-capacitors, and many more devices. Its two dimensional nature gives tremendous sensitivity to its surroundings which increases its applicable nature. However. this sensitivity comes with a price. The material surrounding graphene has to be free of defects and impurities for suitable integration with graphene. In most cases this is impossible, as long-range scattering defects are present in all materials. Charge impurities in the substrate, which cause long-range scattering in the graphene sheet, affect the conductivity and the distribution of charge carriers in graphene.

Due to the sensitivity of graphene, the mathematical models describing its properties have to incorporate a wide variety of potential effects and parameters. Having a model which incorporates all physical effects is very difficult and not necessary as some effects may be negligible in certain scenarios. In our talk we will consider different models of a graphene heterostructure by analyzing their respective Green's functions and looking at the changes they cause in the conductivity. In addition we will look at how correlations in the impurities surrounding graphene can change the conductivity qualitatively. We will compare our theoretical results from the different models with recent experimental results to show correlation between theory and experiment and test the validity of our models.

Vivian Akpene Apety, Brock Unviersity.
Infinitesimal Symmetries of a $(1+1)$-dimensional Integrable Hydrodynamic-type System.

Hydrodynamic-type integrable systems which occur in many physical problems generally appear in $(1+d)$-dimensional homogeneous and non-homogeneous forms. However integrability of hydrodynamic-type systems cannot be studied with the usual standard symmetry approach. In this project, we used some ideas of the symmetry approach to study a known integrable $(1+1)$-dimensional hydrodynamic-type homogeneous system of partial differential equations (PDEs) of order 1. We found the so-called *higher symmetries of arbitrary order* n. Our conjecture is that we have found all higher symmetries for arbitrary order n. We verified this conjecture for small values of n.

Kenneth Blahut, Ryerson University.
Modeling the in vitro spread of hepatitis C virus infections.

Mathematical and computer models that reproduce the spread of a viral infection within a cell culture (in vitro) provide unique, valuable information: the accurate quantification of key infection parameters (e.g., viral production rate, infectious cell lifespan). Changes in these parameters, in turn, can indicate how a mutation affects viral fitness or identify the mode of action and efficacy of novel antiviral drugs. There currently exists only one mathematical model describing the course of a hepatitis C virus (HCV) infection in vitro: a non-spatial ODE model. However, experiments have shown that the spread of HCV infection has an important spatial component: infection disseminates both distally via release and diffusion of virus through the medium, and locally via direct, cell-to-cell infection. Both infection modes appear to play an important role, yet could be differentially affected by antiviral therapy. Therefore, characterizing their relative contribution to infection kinetics has important implications for the control of HCV infections. We have developed a agent-based computer model which explicitly incorporates both distal and local modes of infection. The model consists of a two-dimensional, hexagonal grid in which each site corresponds to one, non-motile, hepatocyte (liver cell). Since experimental measures taken over the course of infection typically report both the concentration of extracellular infectious virus, as well as the count of intracellular viral RNA segments, our model also tracks both of these quantities. Within each cell, the concentration of HCV RNA is tracked and updated via an ODE model for intracellular viral replication. The intracellular concentration within each cell is, in turn, linked to the rates of extracellular release and cell-to-cell infection. In this presentation, I will showcase the range of kinetics exhibited by our model and its performance in reproducing data from experimental in vitro HCV infections.

Christiane Burton, University of Western Ontario.
Theoretical and experimental comparison of x-ray angiographic image quality using energy-based subtraction with conventional subtraction methods.

Purpose: X-ray digital subtraction angiography (DSA) is widely used for vascular imaging. However, subtraction of the mask image from a contrasted image often results in severe motion artifacts. The use of images acquired at two or more x-ray energies (dual energy) has been proposed in the past to produce iodine-specific images. We are investigating the iodine signal and noise ratio (SNR) that can be achieved using both methods and have shown recently that ESA has the potential to provide iodine SNR within 70% to that of DSA for the same patient entrance exposure. In conjunction we are using a cascaded system analysis approach to develop a model for dual-energy in terms of the noise equivalent quanta. The purpose of this study is to provide an experimental validation and comparison of ESA and DSA iodine SNR.

Methods: We have implemented a facility for experimental evaluation of ESA using a vascular phantom consisting of an iodinated step-wedge embedded in 20 cm of water with a fast kV-switching x-ray generator and CsI-based CMOS detector. Optimal energies and high-energy filter to maximize the iodine SNR for a give patient exposure were determined for each method. Image quality was qualitatively compared using a vascular anthropomorphic phantom consisting of iodine-filled tissue-equivalent tubing embedded in a RANDO phantom.

Results: Optimal exposure conditions were determined to be 63 kV for DSA and 50/130 kV for ESA using 2.1 mm Cu to filter the high-energy beam. Both theoretical and experimental results show iodine SNR for ESA is within 30% that of DSA for the same patient entrance exposure. The ESA approach was able to suppress soft-tissue structures in the angiographic image.

Conclusion: Experimental results show excellent agreement with theory. It is concluded that ESA has the potential to produce vascular images with background suppression, similar to DSA, without the need for a mask image and thereby less sensitive to motion artifacts.

Amenda Chow, University of Waterloo.
Control of Hysteresis in the Landau-Lifshitz Equation.

Hysteresis is typically displayed by systems that have multiple stable equilibrium points and dynamics that are faster than the rate at which inputs are varied. One such model is the Landau-Lifshitz equation, a nonlinear PDE which describes the behaviour of magnetization inside a magnetic object. It is known that the Landau-Lifshitz equation has an infinite number of stable equilibrium points. To control the hysteresis arising in this magnetization model requires finding a control that moves the system from one equilibrium to another. The initial equilibrium is no longer an equilibrium of the controlled system and the second point is an asymptotically stable equilibrium point of the controlled system. Such a control for the Landau-Lifshitz equation will be presented.

Lauren DeDieu, McMaster University.
λ-Harmonious Colouring.

In 1983, Hopcroft and Krishnamoorthy designed a new type of graph colouring called harmonious colouring. Harmonious colouring is a proper vertex colouring such that no two edges share the same colour pair. The least number of colours needed to harmoniously colour a graph is called the harmonious chromatic number. We extended this definition of harmonious colouring and defined λ-harmonious colouring, which allows each edge colour pair to occur up to λ times. In this talk, we will examine the results found for the harmonious chromatic number of paths, cycles, and trees. We will also explore λ-harmonious colouring, and I will outline some results we found for the λ-harmonious chromatic number of complete graphs, complete bipartite graphs, paths, cycles, and wheels.

Brian Franczak, University of Guelph.
Model-based clustering and classication via mixtures of shifted asymmetric Laplace (SAL) distributions.

A family of shifted asymmetric Laplace (SAL) distributions called the ParSAL family is introduced and used for model-based clustering. This family of models arises through an eigendecomposition of the component covariance matrices and includes a skewness parameter. An EM algorithm is developed for parameter estimation by exploiting the relationship with the generalized inverse Gaussian distribution and a novel technique for dealing with the issue of "infinite likelihood" is presented. The ParSAL family is applied to both simulated and real data to illustrate clustering applications. In these analyses, our family of mixture models are compared to the popular symmetric approaches. This work concludes with discussion and suggestions for future work.

Clarence D. Kalitsi, Brock University.
Approximate Sampling Distributions of the Parameter Estimators in the AR(1)-model.

We consider the first-order autoregressive model defined by $X_{i+1} = \rho X_i + \varepsilon_i + 1$. Based on a fully analytical approach, we demonstrate how to obtain the first four moments of some well-known estimators of ρ. This enables us to utilize the Edgeworth expansion series to approximate the corresponding sampling distributions, which vastly improves that of the central limit theorem. More importantly, the resulting approximate sampling distributions perform very well even when the sample size is relatively small. In the case of the maximum likelihood estimator of ρ, we further show how this technique can be extended to higher order autoregressive models.

Richard Kohar, Royal Military College.
Optimization of a Wireless Channel.

Given that there are n nodes each competing for access to a single wireless channel, if there are two or more nodes that try to broadcast simultaneously, then this results in a collision, garbled data, and wasted airtime. In every round, the nodes try to access the wireless link on a time slot basis with a predetermined probability, resulting in a distribution of user transmission over slots that is used for contention resolution. Using function component analysis, we will go through a proof to find the global optimum for the case of three slots.

Jeremy Levick, University of Guelph.
The Four-Dimensional Perfect Mirsky Conjecture.

A square matrix with non-negative entries, all of whose rows and columns sum to 1 is called a doubly stochastic matrix. The set of such matrices of size $n \times n$ is denoted Ω_n. Doubly stochastic matrices are closely tied to majorization, a partial order on vectors in $\mathbb{R}^n$, a connection made explicit by the Hardy-Littlewood-Polya theorem. Majorization plays an important role in economics, where it is used to define the Lorenz order, a measure of how equitably distributed wealth is throughout a population, and in quantum mechanics, where it can be used to compare entanglement of two quantum states. Doubly stochastic matrices are therefore a subclass of matrices whose properties are of considerable interest, in many different domains.
In 1965, Perfect and Mirsky conjectured that the region of all possible eigenvalues of all $n \times n$ doubly stochastic matrices (denoted ω_n) would be the union of the regions Π_k for $k \in \{1, 2, \cdots, n\}$, where Π_k is the convex hull of the k^{th} roots of unity. They proved the conjecture for $n = 1, 2, 3$. In 2007, Rivard and Mashreghi exhibited a counterexample for $n = 5$.
In this paper we prove the Perfect-Mirsky conjecture for $n = 4$, and provide a new conjecture for which Rivard and Mashreghi's example is not a counterexample. We discuss some geometric interpretations of the problem of characterizing ω_n, and give some heuristic arguments to support our conjecture.

Cameron McGuinness, University of Guelph.
An Introduction to Monte Carlo Tree Search.

In 2008, a new technique in game AI was introduced, Monte Carlo Tree Search (MCTS). Since then it has garnered much attention in the games research area. It has been used for world champion computer GO players, world champion general game players and many other applications. This talk will introduce MCTS and review some of the improvements and applications of the technique.

Colin Phipps, University of Waterloo.
Investigating the role of interstitial fluid pressure on angiogenic behaviour in solid tumours.

A mathematical model is presented for the concentrations of proangiogenic and antiangiogenic growth factors, along with interstitial fluid pressure (IFP), in solids tumours embedded in host tissue. In addition to diffusive transport, production and degradation of these angiogenic growth factors (AGFs), interstitial convection is included in order to study the locally destabilizing effects of fluid transport on the angiogenic activity endowed by these factors. The molecular sizes of representative AGFs and the outward flow of interstitial fluid in tumors suggest that in many cases convection is a significant mode of transport for these molecules. The resulting balance or imbalance of proangiogenic and antiangiogenic factors serves as a possible mechanism for determining whether blood vessels are stable, developing or regressing. The results of our modeling approach suggest that changes in the physiological parameters that determine interstitial fluid pressure have as profound an impact on tumor angiogenesis as those parameters controlling production, diffusion, and degradation of AGFs. This model has predictive potential for determining the angiogenic behavior of solid tumors and the possible effects of cytotoxic and antiangiogenic therapies on tumor angiogenesis.

Chris Plyley, University of Western Ontario.
Polynomial Identities and the Duality Theorem.

In this talk, I will introduce a well known and powerful duality between actions on an algebra A and group-gradings. Further, I will describe how this duality may be used to answer questions on the polynomial identity structure of A, and we will also see how this duality may be extended to include more general actions and gradings.

Kazi Rahman, University of Guelph.
Dual species model for spatially structured population.

We derive a spacially stuctured polpulation model containing self- and cross- diffusion from ecological point of view. Starting point is the continuous time, discrete space master equation that is frequently used in Theoretical Ecology. The continuous model is obtained by passing to the continuous limit in space. This model is a multi-species generalisation of an existing single species density- dependent diffusion-reaction model. A simulation results is also been carried out to investigating the local mixing behaviour and the effects of their interaction globally.

Vardayani Ratti, University of Guelph.
A mathematical model of honeybees and their diseases.

The western honeybees are vanishing. Recent years have seen honeybees in distress, with up to 35% of colonies breaking down annually. These losses are usually associated with the Colony Collapse Disorder or wintering losses. The main culprits causing colony collapse are the varroa mites and the deadly viruses they carry. In this talk, we present a mathematical model for the honeybees- varroa mites-virus complex. The model consists of ordinary differential equations and focuses on Acute bee paralysis virus only. The model is studied with analytical and computational techniques. Various control strategies are also presented that can be adopted in order to eradicate the disease. It is taken into account that the coefficients are periodic functions in time and Floquet theory is used for the mathematical analysis of the model.

Nick Rogers, McMaster University.
Compelling Interactive Simulations: Stories from the Front of Graduate Teaching and Research in Partial Differential Equations.

In this talk, I will examine the interaction between my current research focus (the partial differential equations of mathematical physics) and my teaching duties. As an offshoot of my current research on the shallow water equations, I developed a novel teaching tool for use in classes on beginning and advanced PDEs. Simulations of various physical processes (eg - heat diffusion) are carried out on touch screen devices in order to allow students to interact with the underlying mathematics in a tactile way. Reactions from students who have experimented with the simulations have been overwhelmingly positive. I will discuss how insights from this pedagogical exercise have affected and shaped the course of my graduate research and led to unexpected overlap and insight.

Monica Wong, University of Guelph.
Parsimonious Biclustering of Gene Expression Data with Applications in Nutrigenomics.

A family of Gaussian mixture models for the biclustering of high-dimensional gene expression data is introduced. Previously, a biclustering algorithm was introduced whereby the mixtures of factor analyzers model was extended to include a binary and row stochastic factor loadings matrix. This particular form of factor loadings matrix resulted in a block-diagonal covariance matrix. Our family is derived by imposing a factor analysis covariance structure and subsequently imposing constraints thereon. Parameter estimates are obtained through a variant of the expectation-maximization algorithm and model selection is performed via the Bayesian information criterion.