2015 Southwestern Ontario Graduate Mathematics and Statistics Conference

May 19-20, 2015

UNIVERSITY of GUELPH

2015 Southwestern Ontario Graduate Mathematics and Statistics Conference

Schedule of Talks

Tuesday, May 19, 2015.

8:30 **Registration and Coffee**

9:00 **Carolyn Augusta**, University of Guelph.
An introduction to variational inference.

9:30 **Mohammad Tavalla**, York University.
The Structure of Test Functions that Identify Weighted Composition Operators.

10:00 **Krishan Rajaratnam**, University of Waterloo.
Orthogonal separation of the Hamilton-Jacobi equation on Spaces of Constant Curvature.

10:30 **Coffee Break**

10:45 **Jeremy Levick**, University of Guelph.
Private Quantum Channels and Quasiorthogonal Subalgebras.

11:15 **John M. Campbell**, University of Waterloo.
Rectangular Schur functions and ribbon Schur functions.

11:45 **Kenneth Blahut**, Ryerson University.
Modeling the spatial spread of Hepatitis C virus infection in vitro.

12:15 **Picture and Lunch**

1:15 **Sierra Gillis**, University of Guelph.
Ring Optimization with Extinction.

1:45 **Lu Lu**, Beijing Institute of Technology.
Dynamic Boundary Stabilization of Schrdinger Equation through a Kelvin-Voigt Damped Wave Equation.

2:15 **Lauren DeDieu**, McMaster University.
Newton-Okounkov Bodies of Peterson and Bott-Samelson Varieties.

2:45 **Coffee Break**

3:00 **Matthew Veres**, University of Guelph.
Deep Learning Architectures for Soil Property Prediction.

3:30 **Drew Neish**, University of Guelph.
Cluster analysis of microbial metagenomic data via mixtures of Dirichlet-multinomial regression models.

4:00 **CMS Social, Brass Taps, 2nd Floor University Centre**

Wednesday, May 20, 2015.

8:30 **Registration and Coffee**

9:00 **Michael Yodzis**, University of Guelph.
Coupling Fishery Dynamics, Human Health and Social Learning in a Model of Fish-borne Pollution Exposure.

9:30 **Melkior Ornik**, University of Toronto.
Optimizing Conference Success: A Problem in Break Scheduling.

10:00 **Patrick Dornian**, University of Waterloo.
An introduction to enumerating flags of a partially ordered set.

10:30 **Coffee Break**

10:45 **Connor Paddock**, University of Guelph.
Highly Symmetric Configurations and Entangled Symmetric Multipartite Qubit States.

11:15 **Marius Oltean**, University of Waterloo.
Geoids in General Relativity.

11:45 **Eric Jalbert**, University of Guelph.
Comparing Semi-Implicit and Full-Implicit Method for Solving Stiff Density Dependent Diffusion-Reaction Equations Arising in Biofilm Growth Models.

12:15 **Picture and Closing Remarks**

2015 Southwestern Ontario Graduate Mathematics and Statistics Conference (SOGMSC'15)

Abstracts

Carolyn Augusta, University of Guelph.
An introduction to variational inference.

In statistical inference, we are interested in making conclusions about a population based on a sample. Often, the sample consists of some data from a process. By combining a likelihood (a probability of seeing the data, under a certain model) and a prior belief (a prior distribution) according to Bayes' Theorem, we obtain the posterior distribution of the model parameters given the data. By examining this posterior distribution, we can make conclusions about, for example, how inaccurate our model is. However, in many applications, the posterior distribution is intractable. Many techniques have been developed to, for example, sample from the posterior distribution (as in Markov chain Monte Carlo techniques), and to approximate the posterior (via variational methods). In variational inference, the intractable posterior distribution is approximated by a known function, and the difference between the approximating distribution and the true posterior distribution is minimized. Inferences are then drawn based on the approximating distribution. In this tutorial-like presentation, the idea behind variational inference will be discussed, and important results will be derived alongside examples.

S.Mohammad Tavalla, York University.
The Structure of Test Functions that Identify Weighted Composition Operators.

In 1964, Forelli, proved that for $p \neq 2$ in the range of $1 \leq p < \infty$ every isometry of $H^p(\mathbb{D})$ into itself is a weighted composition operator. The Hilbert space structure of $H^2(\mathbb{D})$, however, allows many additional isometries.
Very recently, Prof. Gibson form York University and Prof. Lamoureux from University of Calgary abandoned isometries and considered outer functions to show that any linear operator on $H^p(\mathbb{D})$ that maps the class of outer functions into itself is necessarily a weighted composition operator. On the other hand, geophysical imaging entails determining the (generally unknown) operator that models the transmission of an acoustic signal through a section of layered earth. This operator has a very special property: it maps minimum phase signals to delayed minimum phase signals. (A function in $L^2(\mathbb{R}_+)$ is defined to be minimum phase if its Fourier-Laplace transform, which belongs to the Hardy space $H^2(\mathbb{C}_+)$, is outer.) Moreover, Prof. Gibson and Prof. Lamoureux in a separate paper showed that these operators are exactly conjugate via the Fourier-Laplace transform to a weighted composition operator on the half plane $\mathbb{C}_+$. They also showed that these operators can be identified by test functions f, f, where $f(t) = te^t$ is in $L^2(\mathbb{R}_+)$. But the question was:
What other sets of test functions suffice to identify the class of delayed minimum phase preserving operators?
In this paper this problem is completely resolved in the space $\mathcal{A}$ of all analytic functions on the unit disc $\mathbb{D}$, with less technicalities than working in $H^2(\mathbb{D})$.

Krishan Rajaratnam, University of Waterloo.
Orthogonal separation of the Hamilton-Jacobi equation on Spaces of Constant Curvature.

What is in common between the Kepler problem, a Hydrogen atom and a rotating black-hole? These systems are described by different physical theories, but much information about them can be obtained by separating an appropriate Hamilton-Jacobi equation.

Separation of variables is an old but powerful tool for obtaining exact solutions to these equations. Recently, we constructed a new geometric theory of orthogonal separable coordinates for the Hamilton-Jacobi equation on spaces of constant curvature. These coordinates are important for mathematical physics because they separate the Helmholtz equation in Euclidean space or the Klein-Gordon equation in Minkowski space, for example. I will present this theory using diagrams and carefully chosen examples.

Jeremy Levick, University of Guelph.
Private Quantum Channels and Quasiorthogonal Subalgebras.

An important question in quantum information is when a quantum channel can be used to send private information. We examine conditions under which a quantum channel admits a private subsystem, and show a relationship to the operator-algebraic notion of quasiorthogonality of two subalgebras.

John M. Campbell, University of Waterloo.
Rectangular Schur functions and ribbon Schur functions.

Although there is a canonical way of evaluating a Schur function as a linear combination of ribbon Schur functions using the ribbon basis of the algebra of non-commutative symmetric functions, determining the coefficients arising in such an expansion is generally nontrivial. Using a simple sign-reversing involution, we prove a surprising analogue of the Jacobi-Trudi formula which explicitly evaluates an arbitrary Schur-rectangle as a linear combination of ribbon Schur functions, and a non-commutative analogue of this formula in terms of the immaculate basis of the algebra of non-commutative symmetric functions and the ribbon basis of this algebra. We provide an elegant combinatorial proof of a new formula for Schur functions of the form $s_{(2^n)}$.

Kenneth Blahut, Ryerson University.
Modeling the spatial spread of Hepatitis C virus infection in vitro.

Mathematical and computer models that reproduce the spread of a viral infection within a cell culture (in vitro) provide unique, valuable information: the accurate quantification of key infection parameters (e.g., viral production rate, infectious cell lifespan). Changes in these parameters, in turn, can indicate how a mutation affects viral fitness or identify the mode of action and efficacy of novel antiviral drugs. There currently exists only one mathematical model describing the course of a hepatitis C virus (HCV) infection in vitro: a non-spatial ODE model. However, experiments have shown that the spread of HCV infection has an important spatial component: infection disseminates both distally via release and diffusion of virus through the medium, and locally via direct, cell-to-cell infection. Both infection modes appear to play an important role, yet could be differentially affected by antiviral therapy. Therefore, characterizing their relative contribution to infection kinetics has important implications for the control of HCV infections. We have developed a agent-based computer model which explicitly incorporates both distal and local modes of infection. The model consists of a two-dimensional, hexagonal grid in which each site corresponds to one, non-motile, hepatocyte (liver cell). Since experimental measures taken over the course of infection typically report both the concentration of extracellular infectious virus, as well as the count of intracellular viral RNA segments, our model also tracks both of these quantities. Within each cell, the concentration of HCV RNA is tracked and updated via an ODE model for intracellular viral replication. The intracellular concentration within each cell is, in turn, linked to the rates of extracellular release and cell-to-cell infection. In this presentation, I will showcase the range of kinetics exhibited by our model and its performance in reproducing data from experimental in vitro HCV infections.

Sierra Gillis, University of Guelph.
Ring Optimization with Extinction.

Extinction is a natural process that drives biological evolution. In this study the impact of introducing extinction operators into ring optimization was examined. Ring optimizers are spatially structured evolutionary optimizers inspired by the biological phenomenon of a ring species. A small initial population is introduced into a ring-structured space and spreads, using the spatial structure to manage the exploration/exploitation trade-off of the algorithm. Extinction operators eliminate a substantial fraction of the current population, in effect resetting the algorithm to a more exploratory state. Two types of extinction operators are tested and compared. The deluge operator removes population members with lower fitness while the asteroid operator removes population members in a contiguous block of the ring. Three benchmark functions were used, one a discrete simulation and the other two open-ended continuous real functions. The behavior of the extinction operators are different for each of the benchmark functions. The differences in behavior of the extinction operators are explained in terms of the fitness landscapes of the benchmark functions.

Lu Lu, Beijing Institute of Technology.

Dynamic Boundary Stabilization of Schrdinger Equation through a Kelvin-Voigt Damped Wave Equation.

In this paper, we present a dynamic boundary feedback controller (DBFC) for a Schrödinger equation, where DBFC is a wave equation with Kelvin-Voigt damping in order to stabilize the Schrödinger equation. The coupled Schrödinger-wave system (as shown in Fig.1.) is written as follows:

$$\begin{cases} y_t(x,t) + iy_{xx}(x,t) = 0, & 0 < x < 1, t > 0, \\ z_{tt}(x,t) - z_{xx}(x,t) - \alpha z_{xxt}(x,t) = 0, & 1 < x < 2, t > 0, \\ y(0,t) = z(2,t) = 0, & t \geq 0, \\ y(1,t) = kz_t(1,t), & t \geq 0, \\ \alpha z_{xt}(1,t) + z_x(1,t) = -iky_x(1,t), & t \geq 0, \end{cases}$$

where $\alpha > 0$, $k \neq 0$. The two equations are coupled at $x = 1$ with interconnected conditions and fixed at each end.

We first set up the system operator and show it generates a C_0-semigroup of contractions, and the system is well-posed. By detailed spectral analysis, we obtain that the residual spectrum is empty and the continuous spectrum contains only one negative point. Moreover, all the eigenvalues of the system lie in the open left half plane, and their real parts approach negative infinity, by giving the asymptotic expressions of the eigenvalues. Therefore, this controller design moves the eigenvalues of the Schrödinger and wave equations into the second quadrant. It follows that the C_0-semigroup generated by the system operator achieves strong stability.

Lauren DeDieu, McMaster University.
Newton-Okounkov Bodies of Peterson and Bott-Samelson Varieties.

The study of toric varieties is a beautiful part of algebraic geometry. It is an old and active area of research, and has connections with polyhedral geometry, commutative algebra, combinatorics, and symplectic geometry. Its elegant structure also makes it an invaluable tool in other areas of research such as coding theory, physics, and algebraic statistics.

The theory of Newton-Okounkov bodies is a generalization of the rich theory of toric varieties; it associates a convex body to an arbitrary variety (equipped with auxiliary data). Although initial steps have been taken for formulating geometric situations under which the Newton-Okounkov body is a rational polytope, there is much that is still unknown. In particular, very few concrete and explicit examples have been computed thus far. During my graduate studies I have been working on explicitly computing Newton-Okounkov bodies of Peterson and Bott-Samelson varieties. These varieties arise, for instance, in the geometric study of representation theory. In this introductory level talk, I plan to motivate why this theory is important.

Matthew Veres, University of Guelph.
Deep Learning Architectures for Soil Property Prediction.

Advances in diffuse reflectance infra-red spec- troscopy measurements have made it possible to estimate a number of functional properties of soil inexpensively and accurately. Core to such techniques are machine learning methods that can map high-dimensional spectra to real-valued outputs. While previous works have considered predicting each property individually using simple regression methods, the correlation structure present in the output variables prompts us to consider methods that can leverage this structure to make more accurate predictions. In this paper, we leverage advances in deep learning architectures, specifically convolutional neural networks and conditional restricted Boltzmann machines for structured output prediction for soil property prediction. We evaluate our methods on two recent spectral datasets, where output soil properties are shown to have a measurable degree of correlation.

Drew Neish, University of Guelph.
Cluster analysis of microbial metagenomic data via mixtures of Dirichlet-multinomial regression models.

Compositional analysis of the human microbiome is made possible through advanced sequencing techniques, where the output consists of abundance of different bacterial taxa in each microbiome sample. Previously, a Dirichlet-multinomial mixture model has been used for modelling such microbial metagenomic data, where each mixture component represents distinct meta-communities that show similar biota compositions. However, identifying the association of environmental/biological covariates with abundance in different meta-communities remains an important problem. Here, a mixture of Dirichlet-multinomial regression models is proposed and illustrated. These models allow for a probabilistic investigation of the relationship between bacterial abundance and environmental/biological covariates within each inferred meta-community.

Mike Yodzis, University of Guelph.
Coupling Fishery Dynamics, Human Health and Social Learning in a Model of Fish-borne Pollution Exposure.

Pollution-induced illnesses are caused by toxicants that result from human activity, and should be entirely preventable. However, social pressures and misperceptions can undermine efforts to limit pollution, and vulnerable populations can remain exposed for decades. This talk presents a human-environment system model for the effects of water pollution on the health and livelihood of a fishing community. Such problems lie at the interface of biological ecosystem dynamics and human behaviour modelling. We incorporate dynamic social feedbacks that determine how effectively the population recognizes the injured and acts to reduce the pollution exposure. The model is motivated by an incident from 1949-1968 in Minamata, Japan where methylmercury effluent from a local factory poisoned fish populations and humans who ate them. We will discuss the conditions that allow for the outbreak of a pollution-induced epidemic, and explore the sensitivity of the model with respect to its parameters.

Melkior Ornik, University of Toronto.
Optimizing Conference Success: A Problem in Break Scheduling.

Scientific conferences seek to provide their participants with maximal learning value within the allocated time. However, the success of that does not merely depend on the quality of the speakers. It also hinges on the vigilance of the audience. Furthermore, while the organizers usually lack in-depth information on the former, attention of the audience can be easily influenced by modifying the break regimen employed during the event. We develop a mathematical model that seeks to measure the success of a one-day scientific conference using the above quantities. The problem of determining an optimal break scheduling strategy thus becomes a problem in optimal control. We will determine several necessary conditions for optimality and provide an algorithm for determining an optimal break regimen. Finally, we will translate our conclusions into non-mathematical terms, as guidelines for conference break organization.

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Patrick Dornian, University of Waterloo.
An introduction to enumerating flags of a partially ordered set.

Given a set equipped with a partial order, a flag is a series of distinct comparable elements. Our motivating example will be a polytope or polytopal complex under face inclusion, in which a flag corresponds to a series of faces contained within one another. We will discuss how to count flags and efficiently encode the information into a non-commutative generating function called the ab-index. Under the additional assumption that the partial order is Eulerian, we may encode its flags in an even more concise function called the cd-index that satisfies a number of interesting properties we will discuss. We will then introduce research I conducted with Eric Katz on the decomposition of such functions applied to subdivisions of polytopal complexes that gives combinatorial proof of a result previously only obtained through algebraic geometry.

Connor Paddock, University of Guelph.
Highly Symmetric Configurations and Entangled Symmetric Multipartite Qubit States.

Marius Oltean, University of Waterloo.
Geoids in General Relativity.

We develop, in the context of general relativity, the notion of a geoid – a surface of constant "gravitational potential". In particular, we show how this idea emerges as a particular choice of a previously proposed, more general and operationally useful construction called a quasilocal frame – that is, a choice of a two-parameter family of curves in spacetime comprising the boundary of the history of a finite spatial volume. We study the geometric properties of these geoid quasilocal frames, and construct solutions for them in some simple spacetimes.

Eric Jalbert, University of Guelph.
Comparing Semi-Implicit and Full-Implicit Method for Solving Stiff Density Dependent Diffusion-Reaction Equations Arising in Biofilm Growth Models.

A comparison of a semi-implicit and a fully-implicit numerical method will be conducted by use of the previously established mathematical model for the growth of This example is chosen specifically because of its high stiffness. The semi-implicit method follows the idea of the semi-implicit Euler method, while the fully-implicit method computes the solutions for multiple iterations during every time step, each iteration using the newly updated values for the recomputation. These two methods show a minute difference in accuracy, the fully-implicit method being mildly more correct. The difference in computational intensity is more pronounced, the semi-implicit method typically performs between 3 and 4 times faster then the fully-implicit. The two values compare nicely and, given the correct situation, both have their uses. Ultimately, the gain in accuracy from the fully-implicit method does generally justify the increase in accuracy; showing the semi-implicit method as a sufficient numerical method for these types of problems.

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