An SEI Model with Age-of-Infection and Immigration

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We consider an SEI model of disease transmission with age-in-class structure for the exposed and infectious classes. The starting point for this work is the model in [1]. To that, we add immigration of individuals into all three classes. In particular, we allow that individuals may enter the exposed or infectious classes, with a positive age-in-class. We get the following equations:

$$\begin{aligned} \frac{dS(t)}{dt} &= W_S - \mu_S S(t) - \int_0^\infty \beta(a) S(t) i(t,a) da \\ \frac{\partial e}{\partial t} + \frac{\partial e}{\partial a} &= W_e(a) - (\nu(a) + \mu_e(a))\, e(t,a) \\ \frac{\partial i}{\partial t} + \frac{\partial i}{\partial a} &= W_i(a) - \mu_i(a) i(t,a), \end{aligned}$$

with boundary conditions

$$\begin{aligned} e(t,0) &= \int_0^\infty \beta(a) S(t) i(t,a) da \\ i(t,0) &= \int_0^\infty \nu(a) e(t,a) da \end{aligned}$$

for $t > 0$. The age-in-class specific immigration rates are given by $W_e(a)$ and $W_i(a)$; other terms are standard.

Due to the immigration of infected individuals, there is no disease-free equilibrium and hence there is no basic reproduction number. Elimination of the disease is impossible under the model assumptions. The only equilibrium is a unique endemic equilibrium.

By proving certain results on boundedness and asymptotic smoothness of the flow, we establish the existence of an attractor. Then, using a Lyapunov functional, we prove global asymptotic stability of the endemic equilibrium.

The model and its results are applicable to the analysis of an infectious disease within a single country or region. The model explicitly accounts for the fact that in the modern world all countries are connected and therefore disease is continually travelling across boundaries. We reach the conclusion that local control of a disease requires global action.

References

[1] C. McCluskey, *Global stability for an SEI epidemiological model with continuous age-structure in the exposed and infectious classes*, Math. Biosci. Eng. **9**, pp. 819–841 (2012).